

The Market for Information and the Origin of Financial Intermediation

FRANKLIN ALLEN*

The Wharton School, University of Pennsylvania, Philadelphia, Pennsylvania 19104

When information is sold, there is often a reliability problem since anyone can claim to have superior knowledge. Optimal strategies which allow a seller to establish that he is informed are considered in a standard one-period, two-asset model. When risk aversion is unobservable, an information market is viable and both the seller and the buyers are better off participating. However, the seller cannot obtain the full value of his information because of the reliability problem. This provides an opportunity for intermediation since an intermediary may be able to capture some of the remaining value. *Journal of Economic Literature* Classification Numbers: 020, 310, 520. © 1990 Academic Press, Inc.

1. INTRODUCTION

Hirshleifer (1971) suggested that one way individuals might profit from private information about the future returns to securities would be to sell it. However, he did not pursue this possibility, pointing to a reliability problem when information is sold (p. 565):

. . . it may not be possible for an informed individual to authenticate possession of valuable foreknowledge for resale purposes. After all anyone could claim to have such knowledge.

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Leland and Pyle (1977) argued that this problem could be overcome by intermediation. An intermediary could signal its informed status by investing its wealth in assets about which it has special knowledge. Uninformed parties would not find it worthwhile to imitate because of the risk. The problem of resale is also avoided by this arrangement since the intermediary does not need to reveal its portfolio. This insight leads to a theory of intermediation independent of transaction cost considerations.

The present paper considers feasible and optimal strategies for the direct sale of information when buyers cannot observe whether or not the seller is informed. A standard one-period, two-asset model with many traders is used. The returns to the risky asset are assumed to be normally distributed and traders are characterized with heterogeneous exponential utility functions. In addition, the transmission of information from a seller to a buyer is assumed to require a finite time. This restricts the possibilities for resale. Initially, it is assumed that information can be sold only once; this assumption is relaxed later when financial intermediaries are introduced.

An informed person observes a particular signal about the return to the risky asset. Before receiving the signal, the informed person describes a portfolio he will invest in corresponding to each possible signal, together with a payment he will receive from the buyers of the information. Each portfolio and corresponding payment are chosen so that no uninformed person would want them; an uninformed seller would always be better off just investing in the two assets. The set of portfolios and payments also ensures that the informed person correctly reveals the signal he observes. The buyers, each of whom pays an equal share of the total payment, then decide whether or not the information is worth purchasing. Provided buyers are sufficiently numerous, the average cost will be small enough to warrant purchase. Finally, the seller observes and communicates the signal determining his portfolio and the payment received for the information.

In the benchmark case where the reliability of the seller's information is not in question, each buyer can be charged a positive amount, so the seller would choose to sell to a very large number of agents. If the seller's information is in doubt, but his risk aversion is observable, he can demonstrate that he is informed and again sell to a very large number of agents. However, if the seller's risk aversion is unobservable, only part of the value of the information can be captured because it is only worth selling to a small number of agents. Nevertheless, the seller is better off doing this than just investing in the two assets. Thus, an information market is sustained.

The model is extended to the case where time is sufficient for a buyer to resell his information. As previously, the buyer will be better off selling

his information rather than just investing. There is thus a motive for intermediation independent of transaction costs; although the original seller is unable to obtain the full value of his information, the existence of resale possibilities allows an intermediary to extract some of the remaining uncaptured returns. When risk aversion can take any positive value, this intermediation is profitable just because the risk-bearing capacity of the sellers is increased; it is equivalent to making the original seller less risk averse. However, when there is a positive lower bound on risk aversion, it is also profitable for risk-tolerant agents because they can more readily demonstrate that they are informed. This theory contrasts with Leland and Pyle (1977) where part of the justification of intermediation is to prevent resale; here intermediation occurs because resale permits the appropriation of returns to information.

Others have addressed the information reliability problem and its link to financial intermediation. Bhattacharya and Pfleiderer (1985), for example, consider the problem of mutual funds and other institutional investors in ensuring that their portfolios are managed by people with the ability to acquire useful information about security returns. In Campbell and Kracaw (1980), reliability is also ensured by portfolio constraints on the providers of information. Ramakrishnan and Thakor (1984) and Millon and Thakor (1985) examine a model where firms issuing new securities hire information producers, whose effort cannot be perfectly observed, to certify their value. The relationship of these papers to the model here is considered further below.¹

The paper proceeds as follows. Section 2 describes the model. Section 3 considers the case where it is possible to directly observe whether or not a seller is informed and Section 4 examines the case where this is not possible. In Section 5 the form of the optimal contract when state-contingent securities are available is analyzed. Section 6 extends the model to give a theory of intermediation. Finally, Section 7 contains concluding remarks.

2. THE MODEL

A competitive one-period, two-asset model is used. Each trader is endowed with (possibly different) initial wealth W_0 . When trade occurs at the beginning of the period each trader buys M of the riskless asset and X of the risky asset. The price of the safe asset is normalized at unity and the price of the risky asset is P . A trader's budget constraint is therefore

¹ Different aspects of information markets are considered in Admati and Pfleiderer (1986, 1987, 1989) and Allen (1986a, 1986b).

$$W_0 = M + PX. \quad (1)$$

At the end of the period, the safe asset yields R and the risky asset yields r which has the unconditional distribution $N(Es, \sigma_r^2)$ with distribution function $F(r)$. A trader's wealth is then

$$W_1 = RM + rX = RW_0 + (r - RP)X. \quad (2)$$

Each person is risk averse and has an exponential utility function which depends on wealth at the end of the period:

$$V(W_1) = -\exp(-aW_1). \quad (3)$$

Initially, it is assumed that the distribution is unbounded above and has positive density for all $a > 0$.

There is an informed person, I , who is the only person to observe a pretrade signal s , where

$$r = s + \varepsilon. \quad (4)$$

The variables s and ε are independent and distributed $N(Es, \sigma_s^2)$ and $N(0, \sigma_\varepsilon^2)$, respectively, so that

$$\sigma_r^2 = \sigma_s^2 + \sigma_\varepsilon^2. \quad (5)$$

The precision of I 's signal is therefore $1/\sigma_s^2$ and the conditional distribution function of r is $F(r|s)$. For simplicity, it is assumed I 's cost of observing s is zero.² Nobody else can observe s at any cost or any other variable correlated with s , except r .

The informed person can use an auditor to verify his assets, trades, and any payments he receives. This auditing information can be revealed to buyers of information without divulging the signal to nonbuyers.

All agents know the structure of the economy. In particular, they know the distribution of r , they are aware that everybody has an exponential utility function, and they know the distribution of a . They also know that there is an informed person.

The number of traders is large enough that the actions of any single trader or small group of traders have no effect on asset prices so that asset markets are competitive. It is also assumed that prices convey no infor-

² If there were a positive cost, the person would have to decide whether it was worthwhile to become informed, and then choose the optimal value of σ_s^2 if a range was possible at different costs. Apart from this, the analysis would be similar.

mation about s provided only a small number of traders become informed through the information market. This assumption can be justified as an approximation to the case where there is a very large number of traders and the impact of any small group is negligible relative to the noise in asset prices (see, e.g., Grossman and Stiglitz, 1980; Hellwig, 1980; Diamond and Verrecchia, 1981); it can be modeled formally by having an infinite number of investors but is assumed directly here for ease of exposition. The purpose of this assumption is to isolate the information market from the market for the risky asset. This means the information seller does not have to take into account the effect of his actions on the price of the risky asset and hence on the value of his information. Admati and Pfleiderer (1986) have looked at the complex issues this link raises in the absence of the reliability problem. The model considered here is a polar opposite with the reliability problem being analyzed in the absence of any interaction between the markets. This separation and the fact that everybody is risk averse imply that the risky asset has a positive risk premium:

$$\zeta = Es - RP > 0. \quad (6)$$

Finally, it is assumed that the transmission of information from seller to buyer takes a finite time. Initially, there is only time for information to be sold once before trade occurs; in Section 6 on intermediation, buyers of information can resell it.

3. INFORMED PERSON IDENTIFIABLE

This section considers the benchmark case where there is no reliability problem. The information seller can directly establish the validity of his information and charge each buyer a fixed fee η . This fee is equal to the buyer's ex ante valuation of the information which is found by comparing his expected utility with the information to that without it.

When uninformed about s , a trader's expected utility is found in the usual way using (2), (3), and the moment generating function for the normal distribution,

$$EV_u = -\exp \left[-a \left(RW_0 + \zeta X_u - \frac{a\sigma_r^2 X_u^2}{2} \right) \right], \quad (7)$$

where X_u is the trader's demand for the risky asset. Demand is then

$$X_u = \frac{\zeta}{a\sigma_r^2}. \quad (8)$$

Substituting back into (7),

$$EV_u = -\exp \left[-a \left(RW_0 + \frac{\zeta^2}{2a\sigma_r^2} \right) \right]. \quad (9)$$

For an informed investor who has observed s , it can be similarly shown that

$$X_i(s) = \frac{s - RP}{a\sigma_\varepsilon^2} \quad (10)$$

and

$$EV_i(s) = -\exp \left\{ -a \left[RW_0 + \frac{(s - RP)^2}{2a\sigma_\varepsilon^2} \right] \right\}. \quad (11)$$

Taking expectation over s ,

$$EV_i = -\exp \left\{ -a \left[RW_0 + \frac{\zeta^2}{2a\sigma_r^2} + \eta \right] \right\}, \quad (12)$$

where

$$\eta = \frac{1}{2a} \log \frac{\sigma_r^2}{\sigma_\varepsilon^2}. \quad (13)$$

Thus η is the ex ante value of the signal to the buyer and is the most he would be prepared to pay for it. It is higher the less risk averse the investor and the more precise the information.

If prices in the asset markets are unaffected by the sale of information, each buyer can be charged η . This implies that the seller would choose to sell to so many traders that the price of the risky asset would necessarily be affected. It will be shown in Section 4B that when the informed person is not identifiable and his risk aversion is unobservable, he will only sell to a limited number of agents and therefore will only capture part of the value of the information.

4. INFORMED PERSON NOT IDENTIFIABLE

Now suppose that the informed person cannot directly establish that he is informed. Hence, it is impossible to charge a fixed fee to buyers since the uninformed would mimic the informed, thereby creating a reliability problem. The only publicly observable variable that is correlated with I 's

information is the payoff to the risky asset r . Consequently, one can establish that he is informed by initially choosing a portfolio for each possible s and by restricting the total receipts for the information in such a way that no uninformed person would find it profitable to mimic. The portfolios and payments also provide an incentive for the seller to correctly reveal s .

The sequence of events in the information market is as follows.

(i) For each possible s , the seller specifies $[X(s), \Phi(s)]$. The first term is the seller's portfolio and means he will invest $X(s)$ in the risky asset and $W_0 - PX(s)$ in the safe asset; the second term is the total payment from all buyers. This set of portfolios and payments is announced to all traders.

(ii) Traders decide whether or not they wish to purchase the information and n people are permitted to buy; each contracts to pay $\Phi(s)/n$.

(iii) The seller observes s and announces $\hat{s}$ to the traders that contracted to buy information, but not to anybody else. This determines the portfolio the seller must buy, $[X(\hat{s}), W_0 - PX(\hat{s})]$, and the corresponding total payments from the buyers, $\Phi(\hat{s})$.

(iv) Asset markets meet.

(v) The payoff to the risky asset r is realized. The seller consumes the payoff to his portfolio and his payment for information: $rX(\hat{s}) + R[W_0 - PX(\hat{s})] + \Phi(\hat{s})$.

Although it is assumed that buyers pay an equal proportionate share, $\Phi(s)/n$, it will be seen below that this is not crucial. All that matters is the level of $\Phi(s)$; how it is divided among buyers is unimportant.

In order to compare utilities from various actions, it is helpful to use certainty equivalents. For ease of notation, it is assumed that $W_0 = 0$. It follows from (9) that the certainty equivalent of being uninformed, for example, is $\zeta^2/2a\sigma_r^2$.

Section 4A considers the case where risk aversion is observable. Similarly to Section 3, it is shown that the seller's actions will be such that the price of the risky asset will necessarily be affected. It is shown in Section 4B, where risk aversion is unobservable, that it is optimal to sell to a limited number of people so that only part of the value of the information can be captured.

A. Risk Aversion Observable

In order to establish the reliability of information, the contract must demonstrate that the seller will become informed. This requires that for every s the portfolio and payment for information specified in the contract would give an uninformed seller a lower utility than if he just invested in the two assets as in (9). For a seller with risk aversion a , it is necessary that the pair $[X(s), \Phi(s)]$ satisfy the condition³

³ It is assumed in the usual way that when indifferent the seller acts in the buyers' interests.

$$-\int_{-\infty}^{\infty} \exp\{-a[X(s)(r - RP) + \Phi(s)]\}dF(r) \leq -\exp\left[-\frac{\zeta^2}{2\sigma_r^2}\right]. \quad (14)$$

This inequality must be satisfied for every s because otherwise an uninformed seller would find it worthwhile to sign the contract and then announce the value of s for which the inequality was not satisfied. Simplifying (14) gives the equivalent form

$$\Phi(s) \leq \frac{\zeta^2}{2a\sigma_r^2} - \zeta X(s) + \frac{a\sigma_r^2 X^2(s)}{2}. \quad (15)$$

The requirement that this condition be satisfied for every s is both necessary and sufficient for the information reliability problem to be overcome. Clearly it is sufficient; any uninformed person who suggested a set which satisfied it for all s would be no better off than if he had just invested in the two assets. To see that it is also necessary, suppose it were not satisfied so that the seller could be either informed or uninformed. Since there is only one informed person but a large number of uninformed people, a buyer's prior that the seller is the informed person would be zero. Buyers are therefore unwilling to purchase information unless the requirement is satisfied.

The seller's problem in choosing each pair $[X(s), \Phi(s)]$ is to

$$\text{Max } EV_I = -\int_{-\infty}^{\infty} \exp\{-a_I[X_I(s)(r - RP) + \Phi(s)]\}dF(r|s) \quad (16)$$

subject to (15).

Using (15) to substitute for Φ in (16) and evaluating the integral it can be seen that utility is unbounded because Φ and hence EV_I is quadratic in X :

$$EV_I = -\exp\left\{-a_I\left[\frac{\zeta^2}{2a_I\sigma_r^2} + (s - Es)X + \frac{a_I\sigma_s^2 X^2}{2}\right]\right\}. \quad (17)$$

Although taking a larger position in the risky asset means the seller bears greater risk, this is more than offset by the potentially higher total charge for information. Thus, it is possible to conclude that if risk aversion is observable, the seller would choose to take such a large position that the price of the risky asset would necessarily be affected.

Another way of interpreting this result is that the buyers can uniquely deduce the seller's information from his portfolio and the payment for information is then like an addition to initial wealth. With this interpretation, it can be seen that the result does not depend on exponential utility; a similar result will hold whenever buyers can uniquely deduce the seller's information from his portfolio.

B. Risk Aversion Unobservable

Now suppose that the risk aversion parameter a is unobservable. The cases where it can take any finite positive value and where there is an upper and lower bound are considered in turn. The information market is analyzed in three stages. First, the optimal contract that ensures that the seller is informed and truthfully reveals his information is considered; it entails selling to a limited number of people so the seller can capture only a part of the value of his information. Second, it is shown that the informed person is better off selling his information with this optimal contract than just investing in the two assets. Finally, it is demonstrated that the contract can be structured so that buyers are willing to purchase the information.

The first result is:

PROPOSITION 1. *When risk aversion is unobservable and can take any finite positive value, the seller's utility is bounded and he can capture only a portion of the value of the information. The optimal set $[X(s), \Phi(s)]$ that ensures that the seller is informed and truthfully reveals his information and the corresponding certainty equivalent depend on whether s is above or below E_s . For $s \geq E_s$,*

$$X(s) = \frac{s - RP}{a_I \sigma_e^2} > 0, \quad (18)$$

$$\Phi(s) = 0, \quad (19)$$

$$C^+(s) = \frac{(s - RP)^2}{2a_I \sigma_e^2}. \quad (20)$$

For $s < E_s$,

$$X(s) = \frac{s - RP - 2\zeta}{a_I \sigma_e^2} < 0, \quad (21)$$

$$\Phi(s) = -2\zeta X(s), \quad (22)$$

$$C^-(s) = \frac{(s - RP - 2\zeta)^2}{2a_I \sigma_e^2}. \quad (23)$$

Proof. The proposition is proved in two steps. The first is to find the values of $[X(s), \Phi(s)]$ which maximize the seller's utility subject only to the constraint that buyers establish that he is informed, and the second is to show that these values give the correct incentives for the seller to truthfully reveal his information to buyers at stage (iii), i.e., $\hat{s} = s$.

To establish that the seller is informed when risk aversion is unobservable, (15) must be satisfied for all possible a , not just a_l . The right-hand side of (15) is strictly convex in a so a minimum value a^* exists; provided (15) is satisfied for a^* , it will be satisfied for all a .

Differentiating the right-hand side of (15) gives

$$a^{*2} = \frac{\zeta^2}{X^2\sigma_r^4}. \quad (24)$$

Since $a^* > 0$, there are two possibilities depending on whether X is positive or negative. For $X > 0$,

$$a^* = \frac{\zeta}{X\sigma_r^2}. \quad (25)$$

Using this in (15) gives (19), so the corresponding values of X and $C^+(s)$ are (18) and (20), which are equivalent to (10) and (11).

The other possibility is that $X < 0$ and

$$a^* = -\frac{\zeta}{X\sigma_r^2}. \quad (26)$$

Using this in (15) gives (22) and then finding the corresponding values of X and $C^-(s)$ gives (21) and (23).

Whether the seller does better having X positive or negative depends on whether $C^+(s)$ is greater or less than $C^-(s)$; it is greater if s is above Es and less otherwise.

It remains to show that the seller has the correct incentives to truthfully reveal his information. His decision problem at stage (iii) having observed s is to choose $\hat{s}$ to maximize his expected utility:

$$\text{Max } EV_I = -\int \exp\{-a[X(\hat{s})(r - RP) + \Phi(\hat{s})\}dF(r|s). \quad (27)$$

Evaluating the integral, it can be seen that the solution is $\hat{s} = s$. The reason is that the constraint (15) is satisfied for every s so that a pair better than $[X(\hat{s}), \Phi(\hat{s})] = [X(s), \Phi(s)]$ cannot be found since otherwise it would have been chosen at stage (i). Q.E.D.

Figure 1 illustrates $\Phi(s)$. For s above Es , X is positive and there cannot be a charge for information because buyers cannot distinguish between

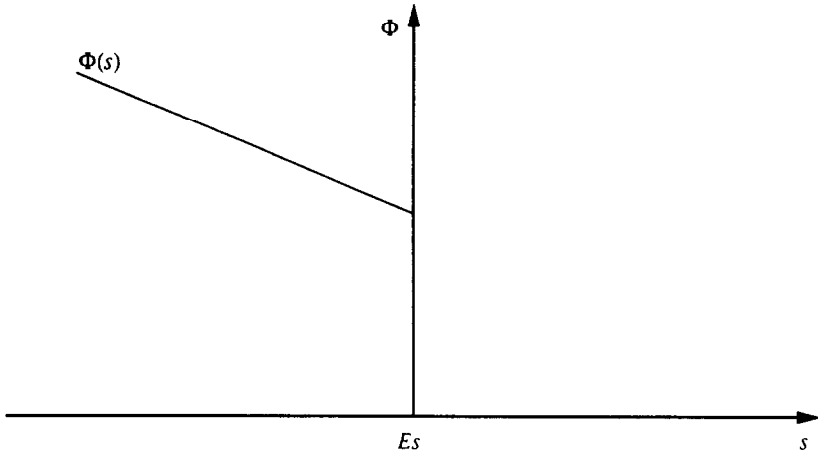


FIG. 1. The total charge for information.

information and risk aversion.⁴ When s is below Es , X is negative and information can be sold at a positive price because uninformed people do not want to take a short position and therefore imitation is costly.

Comparing (21) with (10), it can be seen that the information seller holds less of the risky asset (i.e., a larger short position) than he would if he were just investing. This is because a reduction in X at (10) leads to a second-order reduction in expected utility for an informed person but a first-order reduction for an uninformed person so more can be charged.

The dotted curve in Fig. 2 is an informed person's certainty equivalent from just trading; it has the same form as (20). The solid curve (representing (20) and (23)) illustrates the certainty equivalent of selling information. It is symmetric about Es so that the value of a signal to the seller depends on its deviation from Es , not whether it is good or bad news. When an informed person is just investing in the two assets, his demand for the risky asset switches from negative to positive at $s = RP$, but when selling information it switches at $s = Es$; for s between RP and Es it is better to sell short and charge a positive price for the information than to just take a long position.

It can be seen from Fig. 2 that independent of his information, I is at

⁴ The reason is that an uninformed person has demand $\zeta/a\sigma^2$. Depending on a , this can vary between 0 and ∞ . Hence, it is not feasible to have positive X and at the same time charge anything for information. If a charge were made, there would always be uninformed people who would find it worthwhile to pretend to be informed; in addition to the position they would have taken anyway, they receive a positive payment.

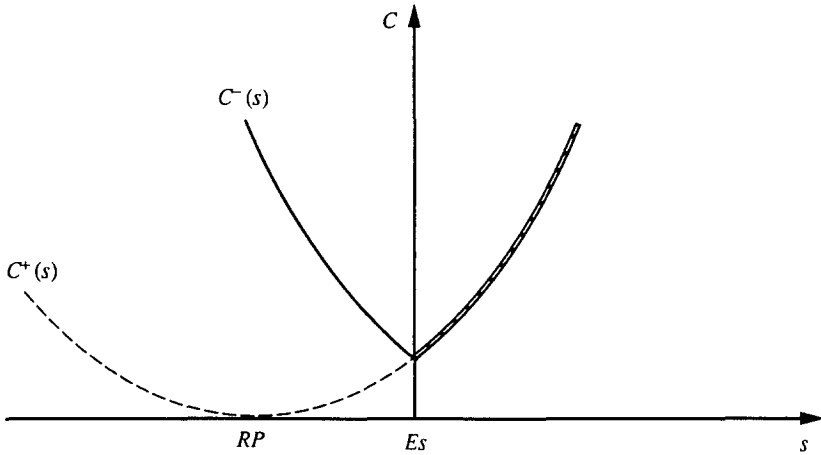


FIG. 2. The informed person's certainty equivalents from selling information and from trading.

least as well off selling information as trading, and for $s < E_s$ he is strictly better off. This gives the second result:

PROPOSITION 2. *The informed person is always better off selling information rather than just investing in the two assets.*

Using (21) and (22) gives that for $s < E_s$

$$\frac{a_I}{\Phi} \frac{\partial \Phi}{\partial a_I} = \frac{\sigma_\varepsilon^2}{\Phi} \frac{\partial \Phi}{\partial \sigma_\varepsilon^2} < 0. \quad (28)$$

It is immediate from (20) and (23) that

$$\frac{a_I}{C} \frac{\partial C}{\partial a_I} = \frac{\sigma_\varepsilon^2}{C} \frac{\partial C}{\partial \sigma_\varepsilon^2} < 0. \quad (29)$$

The less risk averse the seller and the more precise his information, the more he can charge and the better off he will be.

Buyers must have some idea of the information's precision in order to decide whether or not to purchase it at stage (ii). When this precision is not directly observable, it is possible for them to deduce $a_I \sigma_\varepsilon^2$ from the value of $X(E_s) = \zeta / a_I \sigma_\varepsilon^2$ which is revealed at stage (i), but they cannot separately identify a_I and σ_ε^2 .⁵ That is, if a can take any finite positive

⁵ An alternative interpretation is that if the seller were to reveal his precision and his risk aversion at stage (i), he would only have a strict incentive to correctly announce $a_I \sigma_\varepsilon^2$, and would be indifferent about separately identifying a_I and σ_ε^2 .

value, the precision of the seller's information cannot be deduced from the revealed contract. It is shown below that when there is a positive lower bound on risk aversion, it is possible for buyers to deduce an upper bound on σ_e^2 to use in the purchase decision. In Section 5, where state-contingent securities are available, it is possible for buyers to separately identify a_I and σ_e^2 .

Traders will buy information if their expected utility at stage (ii) from buying is greater than that when they are uninformed. To ensure this, it is possible to spread the total payment Φ among enough buyers for the cost to be less than the benefit.

PROPOSITION 3. *There always exists a finite n such that buyers will be prepared to purchase information at stage (ii).*

Proof. See Appendix.

Proposition 3 is derived assuming the information seller can ignore the effect of his actions on the asset markets. If the price of the risky asset conveys information about s to the uninformed, a similar result will hold provided there is enough noise in the supply or demand for the risky asset (see, e.g., Grossman and Stiglitz, 1980; Hellwig, 1980; Diamond and Verrecchia, 1981). In such cases, the information buyers' influence on the price will be sufficiently small that the seller can still choose n large enough to make it worthwhile for them to buy the information. When the noise is insufficient for this, the constraint in the seller's decision problem that buyers have to be better off purchasing information will bind; this introduces risk sharing into the analysis and prevents closed form solutions from being found.

Propositions 2 and 3 together imply:

PROPOSITION 4. *When the degree of risk aversion is unobservable and can take any finite positive value, an information market is sustained.*

Finally, consider the case where risk aversion is unobservable but there is a positive lower bound a_L and a finite upper bound a_U . The counterpart to Proposition 1 is:

PROPOSITION 5. *When risk aversion is unobservable and lies in the range (a_L, a_U) and if $a_I\sigma_e^2 \leq a_L\sigma_r^2$, the seller chooses to sell to so many agents that the price of the risky asset would necessarily be affected. Otherwise, the contract for selling information is the same as that in Proposition 1 except that for s outside the range $s^- = Es - (a_I\sigma_e^2/a_L\sigma_r^2 - 1)\zeta \leq s \leq s^+ = Es + (a_I\sigma_e^2/a_L\sigma_r^2 - 1)\zeta$,*

$$X(s) = \frac{s - Es}{a_I\sigma_e^2 - a_L\sigma_r^2}, \quad (30)$$

$$\Phi(s) = \frac{(s - s^+)^2 a_L \sigma_r^2}{2(a_I \sigma_e^2 - a_L \sigma_r^2)^2}, \quad (31)$$

$$C(s) = \frac{\xi^2}{2a_L \sigma_r^2} + \frac{(s - Es)^2}{2(a_I \sigma_e^2 - a_L \sigma_r^2)}. \quad (32)$$

Proof. See Appendix.

Figure 3 illustrates Φ in this case. It is not possible to charge anything when $a_L = 0$ and $s > Es$ because any positive demand is consistent with being uninformed. When $a_L > 0$, this is no longer true; every uninformed person who chose the $X(s)$ for s outside the range $s^- \leq s \leq s^+$ would be strictly worse off, so information can be sold. For the same reason as that in (21), the seller's demand in (30) is greater in absolute value than an informed person's demand in (10).

In Fig. 4, the solid curve represents the seller's certainty equivalent when $a_L > 0$. For $s^- \leq s \leq s^+$, this is the same as when $a_L = 0$ (the dotted curve represents $C(s)$ for these s). For s outside this range, the seller obtains $C(s)$; this is strictly better than $C^+(s)$ and $C^-(s)$ (represented by the dotted curve for these s) which is what the seller obtains when $a_L = 0$.

Provided $a_I \sigma_e^2 \leq a_L \sigma_r^2$, the counterpart to Proposition 2 also holds. An additional result is that for s outside the range $s^- \leq s \leq s^+$

$$\frac{\partial \Phi}{\partial a_L}; \frac{\partial C}{\partial a_L} > 0; \quad (33)$$

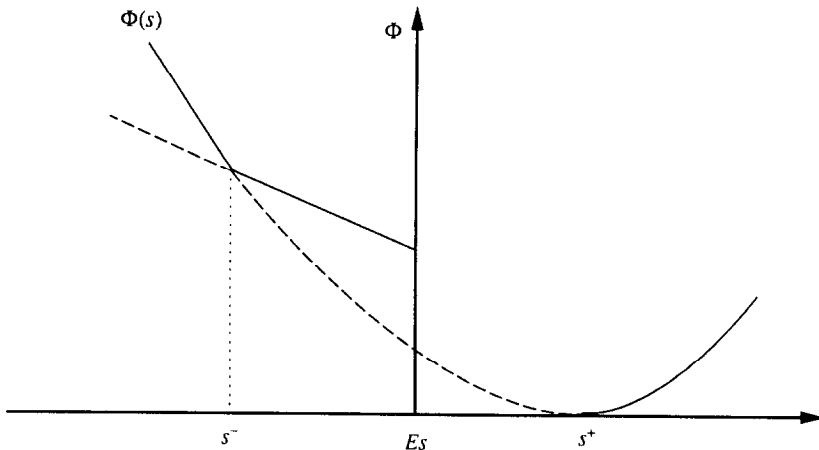
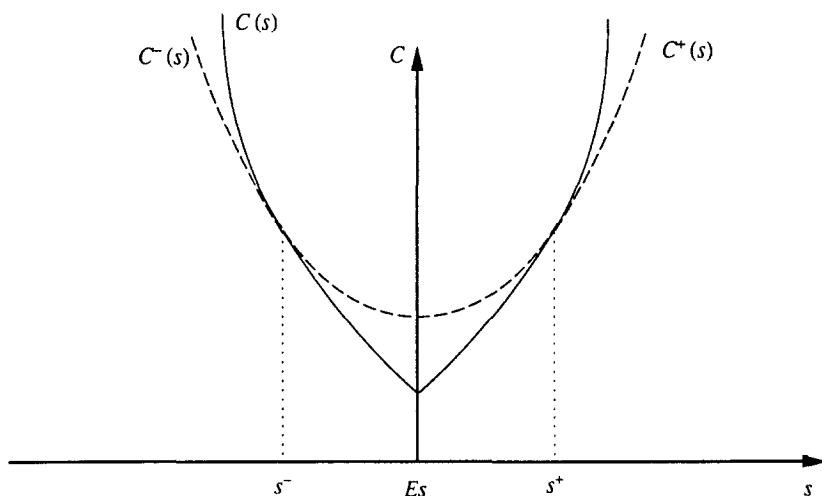


FIG. 3. The total charge for information when $a_L > 0$.

FIG. 4. The certainty equivalent of the seller when $a_L > 0$.

the higher a_L , the nearer s^+ and s^- are to E_s , the more effectively the seller can demonstrate that he is informed and the more he can charge.

From the buyer's point of view the analysis is again similar. The main difference is that it is now possible to deduce an upper bound on the seller's precision. As before, it is possible to find $a_I \sigma_\varepsilon^2$ from the contract specified at stage (i), and combining this with $a_L \leq a_I$ gives $1/\sigma_\varepsilon^2 \leq a_L/a_I \sigma_\varepsilon^2$. If the buyers do not know the seller's actual precision, they can use $a_L/a_I \sigma_\varepsilon^2$ when deciding whether to buy. In this case the seller would need to increase n above the level necessary if his precision were known.

The critical determinant of whether the seller wishes to sell to a limited number of people is the extent to which buyers can distinguish whether a particular portfolio is chosen because of information or because of risk aversion. This will depend on the functional forms of the seller's utility function that are perceived as being possible by the buyers. In general, even though risk aversion is unobservable, the seller may want to sell to so many agents that the price of the risky asset would necessarily be affected.

5. STATE-CONTINGENT SECURITIES

This section considers how information can be sold when securities contingent on r are available. These securities provide one unit of consumption in state r and nothing otherwise; their price is denoted $p(r)$.

It is well known that when agents have exponential utility functions and are symmetrically informed, the allocation of consumption is the same whether safe and risky assets or state-contingent securities are available. Using this result, it can be shown that when r has the distribution $N(Es, \sigma_r^2)$ and the price of the risky asset is P ,

$$Rp(r) = \frac{1}{(2\pi)^{1/2}\sigma_r} \exp \left[-\frac{(r - RP)^2}{2\sigma_r^2} \right]. \quad (34)$$

As above, it is assumed that the actions of the information seller do not affect asset prices, so that (34) is the price function faced by traders.

An uninformed trader's demand for state-contingent securities is

$$x_u(r) = \frac{(Es - RP)}{a\sigma_r^2} (r - RP), \quad (35)$$

and his level of expected utility is the same as that in (9).

An informed trader's demand is

$$x_i(r, s) = \frac{1}{2a} \left\{ -\frac{\sigma_s^2}{\sigma_r^2 \sigma_\varepsilon^2} [r^2 - (RP)^2] + 2 \left(\frac{s}{\sigma_\varepsilon^2} - \frac{RP}{\sigma_r^2} \right) (r - RP) + \frac{\sigma_r^2}{\sigma_\varepsilon^2} \right\}. \quad (36)$$

In contrast to the linear form of (35), this is a quadratic function of r . The informed trader exploits his information by taking a relatively long position in the states where the density function of $N(s, \sigma_\varepsilon^2)$ lies above that of $N(Es, \sigma_r^2)$ and a relatively short position where it lies below. His certainty equivalent is

$$c_i(s) = \frac{(s - RP)^2}{2a\sigma_\varepsilon^2} + \frac{\sigma_s^2}{2a\sigma_\varepsilon^2} - \frac{1}{2a} \log \frac{\sigma_r^2}{\sigma_\varepsilon^2}. \quad (37)$$

The sum of the last two terms is positive, so the value of the information to the trader is strictly increased by the existence of state-contingent securities.

Next, consider the problem of selling information when risk aversion is unobservable and can take any finite positive value as in Section 4B; other cases dealt with in Section 4 can be similarly analyzed. The seller's problem, if he is to identify himself as informed, is to choose his demand for state-contingent securities $x(r, s)$ and the total payment for information $\phi(s)$ to

$$\text{Max} - \int \exp\{-a_I[x(r, s) + \phi(s)]\}dF(r|s) \quad (38)$$

subject to the counterpart of (14) which is

$$- \int \exp\{-a[x(r, s) + \phi(s)]\}dF(r) \leq -\exp\left[-\frac{\xi^2}{2\sigma_r^2}\right] \quad \forall a \quad (39)$$

and his budget constraint.

Similarly to before, the left-hand side of (39) is a concave function of a . Let the maximizing values be a^* , where

$$\int [x(r, s) + \phi(s)] \exp\{-a^*[x(r, s) + \phi(s)]\}dF(r) = 0. \quad (40)$$

The constraint (39) can then be represented by the inequality in (39) with $a = a^*$ and (40). Even though an explicit expression for a^* can no longer be found, it is possible to show:

PROPOSITION 6. *The seller's utility is bounded. The optimal set of $[x(r, s), \phi(s)]$ that ensures that the seller is informed and truthfully reveals his information and the corresponding certainty equivalent are*

$$\begin{aligned} x(r, s) = & \frac{1}{2(a_I - a^*)} \left\{ -\frac{\sigma_s^2}{\sigma_r^2 \sigma_e^2} [r^2 - (RP)^2] \right. \\ & \left. + 2\left(\frac{s}{\sigma_e^2} - \frac{Es}{\sigma_r^2}\right)(r - RP) + \frac{\sigma_r^2}{\sigma_e^2} \right\}, \end{aligned} \quad (41)$$

where a positive value of a^* is given uniquely by

$$\xi_1(a^*) = a^* \sigma_s^2 + \frac{a^{*2} \sigma_r^2 (s - Es)^2}{A} - A \left[\frac{\xi^2}{\sigma_r^2} + \log \left(1 + \frac{a^* \sigma_s^2}{A} \right) \right] = 0 \quad (42)$$

and

$$A = a_I \sigma_e^2 - a^* \sigma_r^2 > 0; \quad (43)$$

$$\begin{aligned} \phi(s) = & \frac{1}{2(a_I - a^*)} \left\{ \frac{a^* \sigma_s^4}{A \sigma_e^2} + \frac{(s - Es)^2}{A} \left[\frac{a^* \sigma_r^2 (a_I - a^*)}{A} + a_I \right] \right. \\ & \left. + \frac{\xi^2}{\sigma_r^2} - \frac{(s - RP)^2}{\sigma_e^2} \right\} > 0, \end{aligned} \quad (44)$$

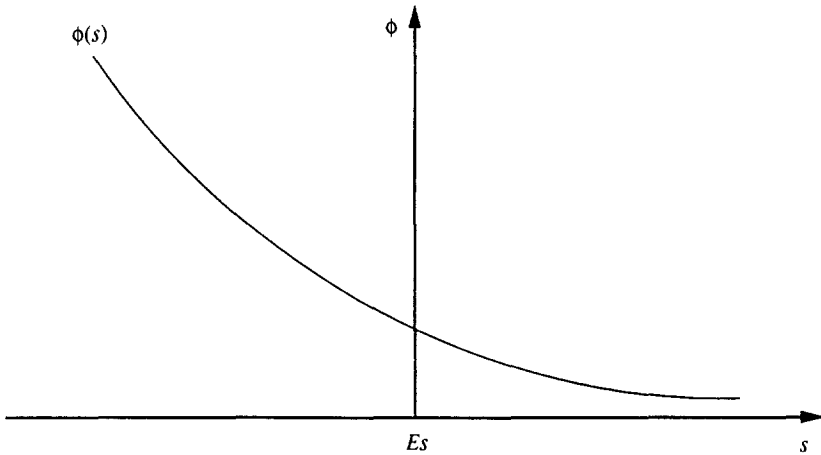


FIG. 5. The total charge for information with state-contingent securities.

$$c(s) = \frac{1}{2} \left[\frac{(s - Es)^2}{A} + \frac{\zeta^2}{a^* \sigma_r^2} + \frac{1}{a^*} \log \left(1 + \frac{a^* \sigma_s^2}{A} \right) - \frac{1}{a_I} \log \left(1 + \frac{a_I \sigma_s^2}{A} \right) \right]. \quad (45)$$

Proof. See Appendix.

In Proposition 1, the information seller did not charge anything for his information when $s > Es$ because any positive demand is consistent with being uninformed; when $s < Es$, the seller charges because it is costly for the uninformed to imitate. In contrast, with state-contingent securities, an uninformed person's demand is a linear function of r , whereas an informed person chooses a quadratic function. Since an uninformed person would always be worse off taking a quadratic position, an information seller can distinguish himself from the uninformed and charge something for his information for all values of s .

Figure 5 illustrates $\phi(s)$.⁶ Although a positive amount can be charged for every s , there is still an asymmetry as in Proposition 1. When an informed person observes a high s , he takes a large long position on average. However, uninformed traders with low risk aversion also want to take a large position, and the cost to them of switching from a linear to a quadratic position is small; thus, an information seller cannot charge very much. For a low s , an informed person takes a short position on average and the cost of imitation is large; thus, the seller can charge a large amount.

⁶ It follows from (42) to (44) that $d\phi/d(s - Es) < 0$, and also that as $s \rightarrow +\infty$, $\phi(s) \rightarrow 0$ and as $s \rightarrow -\infty$, $\phi(s) \rightarrow +\infty$ so $\phi(s)$ has the form shown in Fig. 5.

There are two differences between the informed person's demand in (36) when he is just trading and in (41) when he is selling information. First, $1/2a_I$ is replaced by the larger term $1/2(a_I - a^*)$. As in Proposition 1, the seller takes a more extreme position than when he is just trading because the loss in utility from the greater risk is more than offset by the fact that a higher charge can be made. Second, the coefficient of $r - RP$ switches from $s/\sigma_e^2 - RP/\sigma_r^2$ to $s/\sigma_e^2 - Es/\sigma_r^2$. As in Proposition 1, this is because for $RP\sigma_e^2/\sigma_r^2 < s < Es\sigma_e^2/\sigma_r^2$, it is better to take a short position and increase the amount that can be charged rather than to take a small long position.

The certainty equivalent (45) again depends only on the deviation of s from Es (since (42) implies a^* is symmetric about Es). Also, the seller is better off selling information when state-contingent securities are available than when there are just safe and risky assets. However, the total amount charged is not always higher when there are state-contingent securities because the range of s where the seller switches from a long position to a short position is $RP\sigma_e^2/\sigma_r^2 < s < Es\sigma_e^2/\sigma_r^2$ rather than $RP < s < Es$ (e.g., if $\sigma_e^2 = 0.1$, $\sigma_r^2 = 0.9$, $a_I = 1$, $s = Es = 2$, $RP = 1$ then $\Phi(2) > \phi(2)$).

The other results are similar to those in Section 4B with two exceptions. The first concerns the effect of a change in the seller's precision on the amount that he charges. In Section 4B, the greater the precision, the higher the amount charged. It is shown in the Appendix that $d\phi(s)/d\sigma_e^2$ can have either sign, so that here an increase in the seller's precision can lead to either an increase or a decrease in the amount charged. Then $\phi(s)$ can fall as precision increases (i.e., σ_e^2 decreases) because $Es\sigma_e^2/\sigma_r^2$ decreases and for values of s that cease to be in the range $RP\sigma_e^2/\sigma_r^2 < s < Es\sigma_e^2/\sigma_r^2$, it is no longer worth taking a short position to charge more.

The second result that differs from Section 4B concerns the buyers' ability to deduce the seller's precision. The fact that the seller takes a quadratic position with state-contingent securities means that buyers can separately identify the seller's risk aversion and precision:

PROPOSITION 7. *When state-contingent securities are available, an information buyer can uniquely deduce the seller's precision from the portfolios and payments announced at stage (i).*

Proof. See Appendix.

An alternative interpretation of the model, which is equivalent to the one here, has the seller specify payment schedules of the form

$$y(r, s) = x(r, s) + \phi(s) \quad (46)$$

at stage (i). At stage (iii) he reveals $\hat{s}$ and at stage (v) receives $y(r, \hat{s})$. The

arrangement is equivalent to that above since the buyers can purchase $x(r, s)$ to offset $y(r, s)$ and their net payment is $\phi(s)$ as before.

Bhattacharya and Pfleiderer (1985) use a principal-agent approach to consider how owners of a mutual fund can ensure that only managers with the ability to produce precise estimates of future returns are hired. The owners (the principal) maximize their surplus subject to a reservation utility constraint on the manager (the agent) which is determined by an exogenously given earnings opportunity. The approximately optimal contract when portfolio managers are effectively risk neutral is derived and it is shown that the reward schedule is also a quadratic function of the payoff of the risky asset. Bhattacharya and Pfleiderer obtain a determinate solution even though risk aversion is observable because they assume that there are many potential managers who compete away any rents from their ability. Their results provide an interesting contrast to those obtained here.

6. INTERMEDIATION

It was shown in Sections 4 and 5 that when information cannot be directly verified and risk aversion is unobservable, an information seller can only capture a portion of the value of his information. In this section it is assumed that buyers have sufficient time before asset markets meet to resell the information; they become intermediaries and obtain some of the value of the information that the seller is unable to extract. The model thus provides a theory of intermediation which does not rely on transaction costs.⁷

If the contract used by the original seller and his announcement $\hat{s}$ are observable to second-stage buyers, an intermediary can use this to certify the credibility of the information it sells. Thus, there is no need for the intermediary itself to have a portfolio which signals that it is informed; it can charge a fixed fee as the seller does in Section 3. The other possibility is that the first-stage contract and announcement of $\hat{s}$ cannot be observed by the second-stage buyers and the intermediaries have to certify the credibility of their information in the same way that the original seller does. It follows from Proposition 2 that the intermediaries will always be better off selling the information rather than trading on their own account.

⁷ A financial intermediary is interpreted here as an agent (or agents) that intermediates between an initial seller of information and its ultimate buyer. This is different from the interpretation in other recent papers, such as Diamond (1984) and Ramakrishnan and Thakor (1984), where an intermediary is a coalition of two or more agents.

Hence:

PROPOSITION 8. *If a buyer of information has sufficient time before asset markets meet to act as an intermediary and resell the information, it will always be worthwhile for him to do this.*

Reselling information makes intermediaries better off than just trading because they are able to extract more of the information's value from investors than the original seller can alone. At least part of the reason for this is that increasing the number of sellers increases the amount of risk that can be borne.⁸ The question is whether the risk-bearing capacity of the sellers is the only determinant of the amount that can be extracted. In other words, how is the level of rents that can be extracted by a group related to the level that can be extracted by a single less risk-averse individual with the same risk-bearing capacity?

With exponential utility functions and normally distributed random returns, the risk-bearing capacity of a single individual with risk aversion a is the same as that of a group of individuals, each with risk aversion ma , in the following sense. The certainty equivalent amount required to compensate the single individual for bearing the risk associated with a random return r is the same as the total amount required to compensate the group of m , if they each bear the risk associated with r/m .

First, consider the case in Section 4B where risk aversion can take any finite positive value. It follows from (19) and (22) that the amount charged by a single seller with risk aversion a is the same as that charged by a homogeneous group of sellers, each with risk aversion ma . When state-contingent contracts are available, the same result follows from (42) and (44). Intermediation is profitable because it allows risk to be spread further; intermediaries charging the most are those with the lowest risk aversion, but this is only because of their greater risk-bearing capacity.

Next, consider the case where there is a positive lower bound on risk aversion. The amount that can be charged by a single person is *not* the same as the total for a homogeneous group with the same risk-bearing capacity.⁹ The reason is that the less risk-averse single person can make better use of the lower bound when demonstrating that he is informed than the more risk-averse people in the group. Here the least risk averse

⁸ See the discussion following Proposition 2 in Section 4 where it is shown that less risk-averse sellers charge more for information.

⁹ To see this, note that for the single person, Proposition 5 holds without change. For the group of m people, the expression for s^+ has a_L replaced by a_L/m and (31) implies that the total amount that can be charged, $m\Phi(s)$, is the same as the right-hand side of (31) but with a_L again replaced by a_L/m . It then follows from (33) that the single person can charge more than the group with the same risk-bearing capacity.

can charge the most as intermediaries because of increased risk-bearing capacity, and because they have a comparative advantage in establishing that they are informed. That is, they can announce incentive compatible contracts at least cost to themselves.

PROPOSITION 9. *When risk aversion can take any finite positive value, the total amount that intermediaries charge depends only on their total risk-bearing capacity. However, when there is a positive lower bound on risk aversion, a risk-tolerant individual charges more than a group with the same risk-bearing capacity.*

The actual intermediaries that correspond most closely to those portrayed here are investment management firms, such as mutual funds, which employ analysts to acquire information about stocks. This information is used to make portfolio investment decisions and these portfolios are marketed to investors. The investment management firms are effectively reselling the analysts' information and are subject to strict auditing procedures so that the assumptions in Section 2 concerning the observability of payments are most likely to be satisfied.

Others have examined the information reliability problem.¹⁰ Campbell and Kracaw (1980) analyze a model where reliability is ensured by restrictions on the portfolio choices of information producers. They show that a minimum initial wealth endowment is necessary to ensure reliability, but that this can equally well be provided by an intermediary or an individual. Unlike the model here, intermediation does not allow more to be earned from the information.

In Ramakrishnan and Thakor (1984), firms hire information producers to certify the value of new securities. The utility functions of the information producers are publicly known, and there is a stochastic ex post indicator of the effort expended in acquiring the information. These assumptions enable information reliability to be ensured by conditioning the information producer's payment on the signal of effort. It is shown that when information producers can monitor each other directly, it is best for them to form an infinitely large intermediary rather than operate individually since this permits complete diversification of the risk associated with the effort indicator. Millon and Thakor (1985) extend the analysis by relaxing the assumption that there is costless internal monitoring. Their results are similar except that the resulting intermediary is of finite size. The main difference between these papers and the one here is that utility functions are unobservable and there is no ex post indicator of whether the seller is informed, except the return on the risky asset. Nevertheless,

¹⁰ A somewhat different approach to the role of asymmetric information in explaining intermediation is taken by Chan (1983) and Diamond (1984).

one of the reasons intermediation is profitable is similar; it allows the risk-bearing capacity of the sellers to be expanded.

7. CONCLUDING REMARKS

This paper derives feasible and optimal strategies for selling information. To make the analysis tractable, exponential utility functions and normally distributed returns are assumed. These allow restrictions to be derived which are both necessary and sufficient to ensure the seller is informed. In more general models, simple sufficient conditions are readily found but the same cannot be said for necessary conditions. For example, requiring the expected value of the seller's total receipts in terms of the uninformed distribution to be less than or equal to the amount that would be obtained from investing his initial wealth in the risk-free asset is sufficient to identify the seller as informed, but it is not necessary.

In conclusion, the main results of the paper are the following. First, the reliability problem is not sufficient to rule out the existence of markets where information is sold directly. Second, the operation of such markets depends critically on the information buyers have about the risk aversion of the seller and the securities that are available. Finally, the view of information markets presented leads to a theory of intermediation which is not based on transaction costs. The original seller is able to capture only a portion of his information's value because of the reliability problem; intermediaries make profits because they can capture some of the remaining value.

APPENDIX

Proof of Proposition 3. In order to demonstrate the proposition it is necessary to show that the buyers will be prepared to purchase information if their expected utility at stage (ii) is greater than that when they are uninformed. When a buyer purchases information, the certainty equivalent of being informed conditional on s is

$$C_{iB}(s) = \frac{(s - RP)^2}{a_B \sigma_e^2} - \frac{\Phi(s)}{n} \quad (A1)$$

(where the subscript B refers to the buyer). This is illustrated in Fig. 6 by the solid curve, which is discontinuous at $s = Es$. The dotted curve is the certainty equivalent of being informed at no cost: $(s - RP)^2/a_B \sigma_e^2$. When n

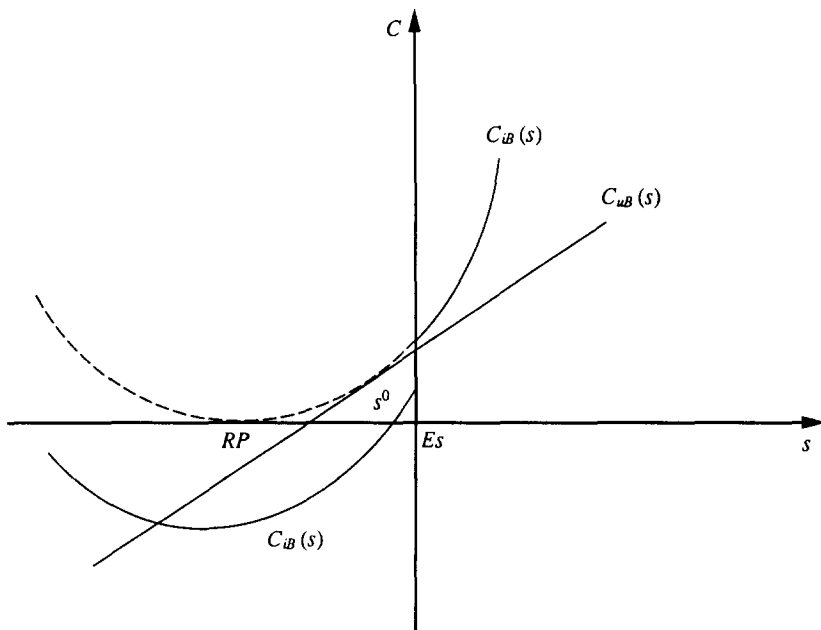


FIG. 6. The buyers' decision.

is increased the effect is to move the $s < Es$ portion of $C_{iB}(s)$ closer to this. In the limit, as $n \rightarrow \infty$, they coincide.

The buyer's demand if he were to remain uninformed is given by (8). This can be used to calculate the certainty equivalent of being uninformed conditional on s as

$$C_{uB}(s) = \frac{(s - RP)\zeta}{a_B \sigma_r^2} - \frac{\zeta^2 \sigma_e^2}{2a_B \sigma_r^4}. \quad (A2)$$

This is illustrated by the solid straight line in Fig. 6. It is tangent to the dotted curve at $s^0 = Es - \zeta \sigma_s^2 / \sigma_r^2$, where $RP < s^0 < Es$, but otherwise lies everywhere below it. For s sufficiently near s^0 , the buyer is worse off ex post from buying information, but for large enough deviations the reverse is true. As n increases, the range of s for which buyers are worse off becomes arbitrarily small so there must be a value where they are prepared to purchase information. Q.E.D.

Proof of Proposition 5. It can be shown using (25), (26), and the appropriate expressions for $X(s)$ from Proposition 1 that $a^* < a_l < a_u$ so the upper bound never binds and has no effect on the analysis.

Again using (25) and (26), it can be shown that for s in the range $s^- \leq s \leq s^+$, $a^* \geq a_L$ so the lower bound on a does not bind and Proposition 1 is relevant. For s outside this range, $a^* < a_L$ so if (15) holds for a_L it holds for all feasible a . Then

$$EV_I = -\exp \left\{ -a_I \left[RW_0 + \frac{\zeta^2}{2a_I\sigma_r^2} + (s - Es)X - (a_I\sigma_e^2 - a_L\sigma_r^2) \frac{X^2}{2} \right] \right\}. \quad (A3)$$

When $a_I\sigma_e^2 \leq a_L\sigma_r^2$, the expression is convex in X as in Section 4A and the seller would choose to sell to so many agents that the price of the risky asset would necessarily be affected. However, when $a_I\sigma_e^2 > a_L\sigma_r^2$ the seller's utility is bounded as in Section 4B and $X(s)$, $\Phi(s)$, and $C(s)$ are as in the proposition.

Proof of Proposition 6. The optimization problem (38) can be solved straightforwardly to give (41) and (44). Substituting these in (40) gives (42). However, (42) alone does not always uniquely define a^* . For example, if $\sigma_r^2 = 2$, $\sigma_s^2 = \sigma_e^2 = 1$, $a_I = 1$, $\zeta = 1$, and $s = Es = 1$, then $a^* = 0.37$ and $a^* = 1.76$ are both solutions to (42). Nevertheless, a unique optimal value of a^* does exist. This is shown in two steps; (i) there exists a unique value of a^* satisfying (42) and (43) and (ii) this value yields a strictly higher expected utility than any other solution of (42) which does not satisfy (43).

(i) A value of a^* satisfying (42) and (43) exists since $\xi_1(0) < 0$ and $\lim_{a^* \rightarrow a_I\sigma_e^2/\sigma_r^2} \xi_1(a^*) > 0$. The value is unique because (43) implies that $\xi'_1(a^*) > 0$ whenever $\xi_1(a^*) = 0$.

(ii) In order for the expected utility integral in (38) to exist (i.e., not to be $-\infty$), either $a^* < a_I\sigma_e^2/\sigma_r^2 (< a_I)$ or $a^* > a_I (> a_I\sigma_e^2/\sigma_r^2)$. To show that $c(s)$ is larger when $a^* < a_I\sigma_e^2/\sigma_r^2$, let

$$\lambda(a) = \frac{1}{a^*} \log(1 + a^*\nu) - \frac{1}{a} \log(1 + a\nu), \quad (A4)$$

where

$$\nu = \sigma_s^2/A. \quad (A5)$$

It is immediate that $\lambda(a^*) = 0$. For $\nu = 0$, it can be shown that $\partial\lambda/\partial a = 0$. For $\nu > 0$, $\partial^2\lambda/\partial a\partial\nu > 0$ and so for $a^* < a_I\sigma_e^2/\sigma_r^2$, $\partial\lambda/\partial a > 0$ and $\lambda(a_I) > 0$. For $\nu < 0$, $\partial^2\lambda/\partial a\partial\nu < 0$ and so for $a^* > a_I$, $\partial\lambda/\partial a > 0$ but now $\lambda(a_I) < 0$.

The remaining terms in (45) are clearly larger when $a^* < a_I \sigma_s^2 / \sigma_r^2$ than when $a^* > a_I$. Hence the solution with a^* satisfying both (42) and (43) gives strictly higher expected utility than any other solution. Q.E.D.

Proof that $d\phi/d\sigma_s^2$ can have either sign. Differentiating (44) and rearranging,

$$\begin{aligned} \frac{d\phi}{d\sigma_s^2} = \frac{1}{a_I - a^*} \left\{ \phi \frac{da^*}{d\sigma_s^2} + \frac{a^* \sigma_s^4}{2A\sigma_s^2} \left[\left(\frac{1}{a^*} + \frac{\sigma_r^2}{A} \right) \frac{da^*}{d\sigma_s^2} - \frac{2}{\sigma_s^2} - \frac{a_I}{A} - \frac{1}{\sigma_s^2} \right] \right. \\ \left. + \frac{a_I(s - Es)^2}{A^3} \left[(a_I - a^*)\sigma_s^2\sigma_r^2 \frac{da^*}{d\sigma_s^2} - (a_I - a^*)a^*\sigma_r^2 - \frac{a_I A}{2} \right] \right. \\ \left. + \frac{(s - RP)^2}{2\sigma_s^4} \right\}, \end{aligned} \quad (A6)$$

where

$$\frac{\sigma_s^2}{a^*} \frac{da^*}{d\sigma_s^2} = 1 + \frac{a^* \sigma_s^2 A^2}{2a_I \sigma_s^2 a^* \sigma_r^2 (a_I - a^*) (s - Es)^2 + a_I a^* \sigma_s^4 A}. \quad (A7)$$

To see that both signs are possible, consider an example with $\sigma_s^2 = 0.9$, $\sigma_s^2 = 0.1$, $a_I = 1$, $Es = 2$, and $RP = 1$. For $s = 1.5$, $d\phi/d\sigma_s^2 = 1.48 > 0$ and for $s = 2$, $d\phi/d\sigma_s^2 = -0.71 < 0$ as required. Q.E.D.

Proof of Proposition 7. To demonstrate the proposition, it is necessary to show that there is a one-to-one correspondence between σ_s^2 and a_I and the set of $[x(r, s), \phi(s)]$ announced at stage (i). It follows directly from Proposition 6 that for each σ_s^2 and a_I there is a unique set, so it remains to show that for each set, the values of σ_s^2 and a_I can be uniquely identified.

To demonstrate this, it is helpful to rewrite (41) in the form

$$x(r, s) = -\alpha r^2 + 2\beta r + \gamma. \quad (A8)$$

Consider what can be deduced given $[x(r, s), \phi(s)]$ for a particular s . Using the definition of α and rearranging,

$$-\frac{(a_I - a^*)\sigma_s^2}{\sigma_s^2} = -\frac{1}{2\alpha\sigma_r^2} = Z, \quad (A9)$$

where $Z (< 0$ from (43)) is a constant which can be found by the buyer.

Using (A9) together with the definition of β allows (42) to be written in the form

$$\begin{aligned} \xi_2(a^*) &= (a^* + Z) \left[\frac{\zeta^2}{\sigma_r^2} + \log \left(\frac{Z}{a^* + Z} \right) \right] \\ &+ a^* - a^{*2} \frac{(\beta/\alpha - Es)^2}{\sigma_r^2(a^* + Z)} = 0. \end{aligned} \quad (\text{A10})$$

Since $\xi_2(0) < 0$, $\lim_{a^* \rightarrow -Z} \xi_2(a^*) > 0$, and $\xi_2'(a^*) > 0$, this equation enables one to determine the unique value of a^* satisfying (42) and (43).

Since from (42) $da^*/d(s - Es)^2 < 0$, it is possible to use $[x(r, s), \phi(s)]$ for two values of s (distinguished by the subscripts 1 and 2) to find values of a_1^* and a_2^* . Using (A9) and the definition of α it can then be shown that

$$\sigma_s^2 = \frac{2\alpha_1(a_I - a_1^*)\sigma_r^2}{1 + 2\alpha_1(a_I - a_1^*)\sigma_r^2} \quad (\text{A11})$$

where

$$a_I = \frac{a_1^* - a_2^*}{\alpha_1 - \alpha_2}. \quad (\text{A12})$$

Q.E.D.

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